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Xiaojun Yang

Local Fractional Integral Transforms

An Introduction to Local Fractional Functional Analysis and Its Applications to Integral Transforms via Local Fractional Calculus

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To my parents, brother Xiao-Bing Yang and sister Xiao-Fang Yang

To my wife Yu-Min Jin and son Bo-Yuan Yang

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Local Fractional Integral Transforms

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Abstract

Over the past ten years, the local fractional calculus revealed to be a useful tool in various areas ranging from fundamental science to various engineering applications, because it can deal with local properties of non-differentiable functions defined on fractional sets. In fractional spaces, a basic theory of number and local fractional continuity of non-differentiable functions are presented, local fractional calculus of real and complex variables is introduced. Some generalized spaces, such as generalized metric spaces, generalized normed linear spaces, generalized Banach's spaces, generalized inner product spaces and generalized Hilbert spaces, are introduced. Elemental introduction to Yang-Fourier transforms, Yang-Laplace transforms, local fractional short time transforms and local fractional continuous wavelet transforms is presented based on local fractional calculus.

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Contents

Preface......6
Acknowledgments......8

***Chapter 1 Preliminaries*......9**
1.1 Sets......9
1.1.1 Some infinite sets......9
1.1.2 The fractional sets of the sets......9
1.1.3 The fractional sets of element sets......10
1.2 Real line number system......11
1.2.1 Real and complex spaces on a fractional set......11
1.2.2 Geometric representation of real line number......13
1.2.3 Operations with real line number......14
1.2.4 Absolute value with real line numbers......16
1.2.5 Inequalities with real line numbers......16
1.3 Point sets and intervals on a fractional set......17
1.4 Countability......18
1.5 Neighborhood on a fractional set......18
1.6 Limit point of fractional set......18
1.7 Bounds on the fractional set......19
1.8 Generalized Lebesgue measure on fractional sets......22
1.9 Generalized Hausdorff measure on fractional sets......23
1.10 Functions on fractional sets......24
1.11 Hausdorff Dimension on fractional sets......26
1.12 Limit of functions......27
1.13 Local fractional continuity of functions......27
1.14 Elementary functions on a fractal set......31
1.15 Special functions on a fractal set......36
1.16 Summery......39

***Chapter 2 Fundamentals of local fractional calculus of real variables*......38**
2.1 Introduction to local fractional calculus......38

2.1.1 Introduction *38*
2.1.2 Birth of local fractional calculus *38*
2.1.3 Historical development of local fractional calculus *39*
2.2 Local fractional derivative *41*
2.3 Application of local fractional derivative *48*
2.3.1 Extreme values *48*
2.3.2 Local extreme value *49*
2.3.3 Critical number *50*
2.4 Local fractional integral *52*
2.4.1 Local fractional integral *52*
2.4.2 Properties of local fractional integral *56*
2.4.3 Theorems of local fractional integral *58*
2.4.4 The local fractional integral of trigonometric functions *63*
2. 5 Local fractional Taylor theorem *66*
2.5.1 Local fractional Taylor theorem *66*
2.5.2 Yang-Taylor's series *68*
2.5.3 Mc-Laurin's series to elementary functions *69*
2.6 Local fractional indefinite integral *69*
2.7 Local fractional differential equations *69*
2.8 The total local fractional differentials *72*
2.8.1 Local fractional partial derivative *72*
2.8.2 Local fractional partial derivative of higher order *73*
2.8.3 The total local fractional differentials *76*
2.8.4 Local fractional derivative of composite function *79*
2.8.5 Local fractional Jacobian determinant *80*
2.9 Chapter Summary *83*

Chapter 3 Fundamentals of local fractional calculus of complex variables* *86
3.1 Motivation of local fractional calculus of complex variable *86*
3.2 Complex function on a fractal set *87*
3.3 Limit of complex function *87*
3.4 Local fractional continuity *88*
3.5 Elementary complex functions *89*
3.6 Local fractional derivatives of complex functions *89*
3.7 Local fractional Cauchy-Riemann equations *91*
3.8 Local fractional integral of complex functions *93*

3.9 Theorems for local fractional integral of complex functions 96

3.10 Local fractional Taylor's series of complex functions 99

3.11 Singular point and Poles 100

3.12 Local fractional Laurent's series 101

3.13 Generalized residue Theroms 104

3.14 Chapter Summary 107

Chapter 4 Generalized fractal spaces 108

4.1 Generalized metric spaces 108

4.1.1 Generalized metric spaces 108

4.1.2 Generalized semi-metric spaces 113

4.1.3 Local fractional continuity, convergence, and completeness in generalized metric spaces.......114

4.2 Generalized normed linear spaces 122

4.2.1Generalized linear spaces 122

4.2.2 Generalized normed linear spaces 123

4.2.3 Generalized linear operators 126

4.3Generalized Banach spaces 129

4.4 Generalized Banach algebra 132

4.5 Generalized inner product spaces 134

4.6 Generalized Hilbert spaces 142

4.7 Fractal orthogonal system 145

4.8 Chapter Summary 146

Chapter 5 Local fractional Fourier series 147

5.1 Motivation of local fractional Fourier series 147

5.2 Local fractional Fourier series 148

5.3 Properties of local fractional Fourier series 150

5.4 Theorems for local fractional series 152

5.5 Expression of local fractional Fourier series 161

5.6 Chapter Summary 163

Chapter 6 Yang- Fourier transforms 164

6.1 Motivation of the Yang- Fourier transforms 164

6.2 Yang-Fourier transforms 165

6.2.1 Yang-Fourier transforms 165

6.2.2 Inverse formula of Yang-Fourier transforms 166

6.3 The convolution 166

6.4 Theorems for Yang-Fourier transforms 167
6.5 The Dirac's distribution 175
6.6 Special functions and their Yanng-Fourier transforms 178
6.7 Table of Yang-Fourier transforms 180
6.8 Heisenberg uncertainty principle for the Yang-Fourier analysis 181
6.9 Applications of Yang-Fourier transforms 184
6.10 Chapter Summary 185

Chapter 7 Yang-Laplace Transforms 186
7.1 Motivation of the Yang-Laplace transforms 186
7.2 Yang-Laplace transforms 187
7.3 Theorems for Yang-Laplace transforms 188
7.4 Convolutions 192
7.5 The Dirac's distributions 119
7.6 Table of Yang-Laplace transform of elementary functions 197
7.7 Applications of Yang-Laplace transforms 198
7.8 Chapter Summary 200

Chapter 8 Local fractional short time transforms 201
8.1 Motivation of local fractional short time transforms 201
8.2 Local fractional short time transforms 202
8.3 Theorems for local fractional short time transforms 203
8.4 Examples for local fractional short time transforms 208
8.5 Chapter Summary 208

Chapter 9 Local fractional continuous wavelet transforms 209
9.1 Motivation of local fractional continuous wavelet transforms 209
9.2 Local fractional continuous wavelet transforms 211
9.3 Theorems for local fractional continuous wavelet transforms 212
9.4 Chapter Summary 220

References 221

Preface

There are many problems in engineering and physics where local fractional derivatives and local fractional integrals play an important role. This paper introduces local fractional functional analysis and some methods of integral transforms based on local fractional calculus, and elucidates a new and mathematically rigorous account of the integral transforms including Yang-Fourier transforms, Yang-Laplace transforms, local fractional short-time continuous transforms and local fractional wavelets transforms. This paper is divided into a number of parts.

Chapter 1 provides basic theory of real line number on fractional sets and proposes the local fractional continuity of non-differentiable functions. The real and complex spaces on fractional sets, the complex conjugation, absolute value, and polar form of a fractional complex numbers are proposed and geometric representation of real line number on fractional sets is discussed. Furthermore, generalized Lebesgue measure, point sets, intervals, countability, neighborhood, limit point, limit of a fractional sequence, bounds, generalized Hausdorff measure and functions on fractional sets are derived. Finally, the theory of local fractional continuity of non-differential functions is discussed and some basic functions on fractional sets are discussed.

In chapter 2 the fundamentals of the local fractional calculus of real variables are outlined. The chapter begins with the definition of local fractional derivatives and the elementary theory of local fractional derivatives of non-differential functions; Topics include the local fractional differential, local fractional of high order, local fractional Rolle's theorems, mean value theorems, local fractional Fermat's theorem, increasing or decreasing test, and derivative test. Existence of the local fractional integrals is proved and the basic properties and theorems of the local fractional integrals are discussed, such as mean value theorem, anti-differentiation, local fractional integration by parts, local fractional Taylor theorem and Yang-Taylor series. To study local fractional differential equations, the concept of the local fractional indefinite integrals is derived. To investigate the total local fractional differentials, local fractional partial derivative, local fractional derivative of high order, and local fractional Jacobian determinant are introduced.

Chapter 3 introduces the fundamentals of the local fractional calculus of complex variables. With the chapter starts by deriving limit and local fractional continuity of complex functions on fractional sets, the local fractional derivatives, local fractional Cauchy-Riemann equations and local fractional integrals of complex functions. The local fractional Cauchy integrals of complex functions and local fractional Taylor's series (also called Yang-Taylor series) and local fractional Laurent's series are also discussed. Lastly, the generalized residue theorems are mentioned. A short outline of local fractional complex analysis is proposed in this chapter.

Chapter 4 derives generalized fractal spaces, such as generalized metric spaces, generalized normed linear spaces, generalized Banach spaces, generalized inner product spaces, and generalized Hilbert spaces. Based on above definitions, we present the completeness of generalized fractal spaces, and extend contracting mapping theorem and generalized contracting mapping theorem, existence and unique of local fractional differential equation, Banach algebra, Pythagorean theorem and the basic criterion for generalized Hilbert spaces to fractional spaces. We obtain the generalized Holder inequality, the generalized Cauchy-schwarz inequality and the generalized Minkowski inequality and some spaces are discussed.

Chapter 5 derives the local fractional series containing fractional trigonometric and Mattag-Leffter forms. Meanwhile, the properties and theorems of the local fractional series are discussed.

Chapter 6 introduces the Yang-Fourier transforms derive from local fractional series based on the local fractional calculus. Meanwhile, the properties and theorems for Yang-Fourier transforms are discussed. In addition, Heisenberg uncertainty principles in fractal spaces are investigated. Applications of the Yang-Fourier transforms to local fractional ordinary differential equations and local fractional ordinary differential systems are taken into account.

Chapter 7 introduces the Yang-Laplace transforms derived from Yang-Fourier transforms. We derive the properties and theorems for the Yang-Laplace transforms and take into account its applications of local fractional ordinary differential equations and local fractional ordinary differential systems.

Chapter 8 studies the local fractional short time transforms, and properties and examples of possible applications.

Chapter 9 derives local fractional continuous wavelet transforms based on the Yang-Fourier transforms and the theorems of the local fractional continuous wavelet transforms.

In this book the various transformations are derived step by step in great detail. We hope this book will be a useful tool for all those who use local fractional integral transforms and local fractional continuous wavelet transform in their work whether they are engineers, financial planners, mathematicians and scientists. The book can also provide a first course on local fractional integral transforms. Several mistakes and misprints were pointed out to us by a number of people and had been corrected. We want to thank those people for their helpful comments. We welcome comments from our readers.