Complex Mechanics

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Abstract

According to the philosophy of Tai Chi, the Nature (the *Tao*) contains two parts: *yang* is the observable (real) part, which constitutes the physical world we experience in daily lives, and *yin* is the unobservable (imaginary) part. As a scientific realization of Yin-Yang philosophy, complex mechanics advocates that the complex space is where a physical quantum motion takes place, while the real space is where we take measurements of the motion. Understanding physical motions in the complex space is helpful to explain the why and the how of the quantum phenomena manifesting in the measurement data taken in the real world. In the complex space, probabilistic viewpoint and deterministic viewpoint become identical such that classical mechanics and quantum mechanics can be unified into the same framework – complex mechanics. A teaching program based on complex mechanics at National Cheng Kung University (NCKU) showed that students enrolled in this course are benefited greatly from the bridge provided by complex mechanics that allows them to accelerate and deepen the learning of quantum mechanics by their previous knowledge and experience gained from engineering mechanics. The success of the NCKU teaching program indicates that complex mechanics shortens the gap between quantum mechanics and classical mechanics and makes the teaching and learning of quantum mechanics much easier. The present book is the outgrowth of the lecture notes of this course. The readers will find in the book that dynamic equations of motion compatible with Schrödinger equation are just the Newton equations extended to a complex domain, by which there will be no ambiguity at all in understanding the motion of quantum particles, and the various interpretations of quantum mechanics are no longer needed. This book is written to be completely self-contained and self-consistent so that readers familiar with Newton mechanics will find that they already possess the ability to handle quantum-mechanical problems even though they did not take a related course of quantum mechanics.

How to Cite this Article ?

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中 文 摘 要

根据太极学说，大自然(道)包含二部分：阳是可观测的部分，是吾人在日常生活中所感知到的物理世界；阴则是不可观测的另一半部。作为一个阴阳学说的严谨科学实现，复数力学主张复数空间是物理运动的原生地，而实数空间则仅是实验数据取得的地方。 明白物理运动如何在复数空间中开展，有助于解开隐藏在实验数据中的量子神秘面纱。在复数空间所建立的沟通桥梁下，波尔的机率诠释与爱因斯坦的因果诠释变得一致，这使得经典力学(牛顿力学)与量子力学可以被统一在复数力学的理论框架下。于国立成功大学(NCKU)的复数力学教学实验显示，修课的学生获益于复数力学所提供的桥梁，使得他们能够藉助先前在工程力学所累积的知识及经验去加速并加深对量子力学的学习。NCKU 教学试验的成功充分证明复数力学有效填补了牛顿力学与量子力学间之理论落差，使得量子力学的教与学均变得更加简易。这本书就是由这一系列的教学讲义所编撰而成。在这本书中，读者将会发现和薛丁格(Schrödinger)波动方程式兼容的动力学原来就是定义在复数空间的牛顿力学，这使得量子运动的描述变得非常明确，而且各种关于量子力学的主观诠释也不再需要。这本书涵盖了学习复数力学所需要的全部信息，读者只要熟悉牛顿力学及复数的基本观念，即具备阅读本书的能力，同时具备了求解量子力学的能力，纵使他们以前不曾学习过量子力学的相关课程。

如何引用该文章?

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Contents

Preface

Complex mechanics is a new mechanics treating the Nature as a complex-valued world wherein all physical quantities are assumed to complex variables. In the complex space, probabilistic viewpoint and deterministic viewpoint become identical so that classical mechanics and quantum mechanics can be unified into the same framework and both can be learned according to the same disciplines.

Consistency between Probabilism and Determinism

The lack of dynamic equations of motion with respect to time, which is one of the sources of controversies of quantum mechanics, makes it impossible to analyze stability, chaos, bifurcation, and many other nonlinear features existing in quantum systems. This impossibility has long been taken for granted due to a common belief that the probabilistic nature of quantum phenomena is in no way described or represented by deterministic nonlinear models. However, probabilistic viewpoint and deterministic viewpoint may not be as conflicting as we commonly think. Consider a scenario that a dynamic motion occurs in the complex space but only its real-part motion can be measured. Due to the influence of the unmeasurable imaginary-part motion and its interaction with the real-part motion, the measured real-part motion is unpredictable and can only be described probabilistically. On the other hand, the same motion, if viewed from the complex space, is governed uniquely by a set of complex-valued nonlinear equations, which are entirely deterministic. In such a situation, probabilistic interpretation and deterministic interpretation can be equally applied to the same motion, depending on which space we deal with.

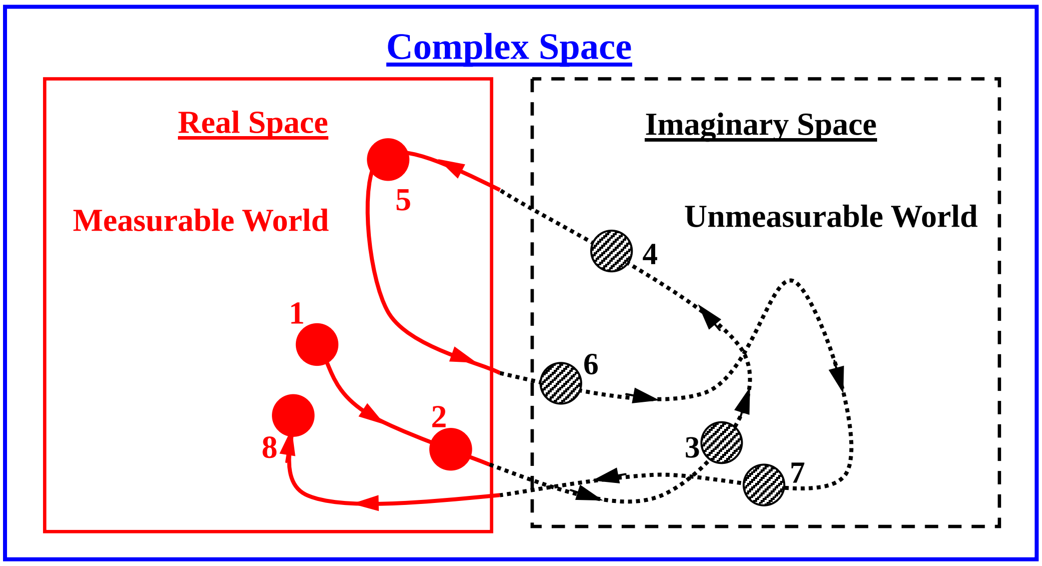

Fig. A. When viewed from the complex space, the motion of a particle is deterministic by following a continuous path $1 \to 2 \to 3 \to \cdots \to 8$, while as viewed from the real space, the particle's motion is discontinuous and causality relation is broken. Because the data points 3, 4, 6 and 7 in the imaginary space are unmeasurable, the causal connection between points 2 and 5 is missing as viewed from the real space, and it appears that the particle jumps randomly from point 2 to point 5 and to point 8.

Figure A is a schematic explanation of why deterministic interpretation and probabilistic interpretation can be applied simultaneously to the same physical motion. Consider a particle moves deterministically in the complex space by following a continuous path $1 \to 2 \to 3 \to \cdots \to 8$. However, the same motion, as viewed from the real space, is discontinuous and does not have any causal connection. Because the data points 3, 4, 6 and 7 appearing in the imaginary space are unmeasurable, the causal connection between points 2 and 5 is missing as viewed from the real space, and it appears that the particle jumps randomly from point 2 to point 5 and then to point 8. In the absence of causal connections between points 2, 5 and 8, it is very natural to treat the particle's position as random variable and use probability to describe its appearance.

In quantum mechanics, we encounter the same scenario that physical motions occur in the complex space, but what we sense and measure are merely the real parts of the motions, which give rise to what we call quantum phenomena. Therefore, understanding the how and the why of quantum mechanics requires a viewpoint from the complex space. The complex space is where a quantum motion takes place, while the real space is where we take measurements of the motion. Quantum mechanics lays out the distribution and the evolution of the measurement data, while the complex mechanics describes the quantum motion in the complex space before it is measured. In the complex space, quantum motions are deterministic so that all the methods of classical mechanics can be applied. The projection of the complex-valued motion into the real space recovers and confirms the quantum phenomena observed from the measurement data. In recent years, an excellent consistency of the projected solutions with the various quantum effects has been justified under the framework of complex mechanics and most of them will be discussed in this book. By the publication of this book, the author wishes to share the delight of teaching and learning complex mechanics with the readers.

Unification of Classical and Quantum Mechanics

Complex mechanics, a unified approach to classical and quantum mechanics, provides a bridge between the probabilistic interpretation in the real space and the deterministic interpretation in the complex space. Through this bridge, researchers in classical mechanics can employ methods familiar to them to analyze quantum systems in the complex space, and to predict and verify various quantum phenomena by projecting the results of complex analysis into the real space. Complex mechanics employs complex-extended Hamilton equations of motion to describe and model quantum systems in such a way that all particle-like properties can be preserved by the classical equations of motion and in the meanwhile, all the wave-like properties are manifested naturally via the multi-path behavior of complex trajectories. By employing the complex-extended Hamilton equations or the equivalent complex Newton equations, there will be no ambiguity at all in understanding the motion of quantum particles; in other words, we no longer need the so-called "interpretation". Everything becomes transparent and quantum mechanical problems can be solved by the well-established methods in the classical mechanics. Figure B highlights the features of complex mechanics, showing that by inputting classical motions to the complex space, we automatically obtain the quantum motions as the output. In other words, the education of quantum mechanics is an integration of engineering mechanics (classical mechanics) and complex variable theory. Therefore, undergraduate students completing the courses of engineering mechanics and complex variable theory (engineering mathematics) have already gathered all the required ability and tools to solve quantum mechanical problems.

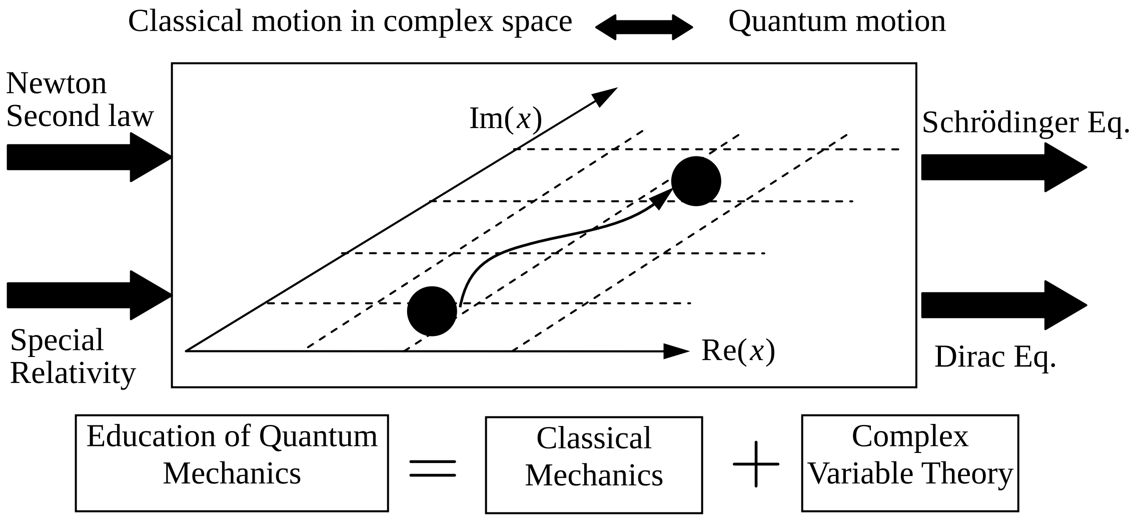

Fig. B. Under the framework of complex mechanics, quantum mechanics is equivalent to classical mechanics defined in the complex domain so that the learning of the former becomes the learning of the latter incorporated with complex variable theory.

Teaching Quantum Mechanics Fully by Engineering Mechanics

With advances in material synthesis and device processing capabilities, the importance of quantum mechanics in material science, electrical engineering and applied physics, has dramatically increased over the last couple of decades. The engineers can no longer just work with simplistic phenomenological equations, but must understand a more fundamental origin of the phenomena. Devices such as Josephson junctions, semiconductor lasers, transistors, and all of the nanostructures cannot be fully understood in terms of simple classical mechanics. The big challenge does not stem from our ignorance of the importance of nourishing engineering education with quantum elements, but from the seemingly insurmountable task of teaching quantum mechanical concept in a class of engineering mechanics. The fundamental ideas of quantum mechanics and engineering mechanics are so conflicting that all the deterministic rules and causal relations found in engineering mechanics are no longer valid in quantum mechanics, which treats physical quantities as random variables having only probabilistic nature. Some lecturers of quantum mechanics even suggest students to forget temporarily the impressions of determinism and causality gained from classical mechanics, when they study quantum mechanics. It appears that the knowledge and experience learned from studying engineering mechanics cannot be conveyed directly to the study of quantum mechanics. These remarkable gaps between quantum mechanics and engineering mechanics have forbidden the possibility to teach quantum mechanics in a course of engineering mechanics or classical mechanics.

To surmount the above-mentioned obstacles, a teaching program based on complex mechanics was launched by the author at National Cheng Kung University (NCKU). This teaching program has been tested in a course named "Engineering Quantum Mechanics" in the Department of Aeronautics and Astronautics at the semester years from 2006 to 2009. The preliminary validation of teaching quantum mechanics fully by the concepts of classical mechanics is very successful in this course. Students enrolled in this course are benefited greatly from the bridge provided by complex mechanics that allows them to accelerate and deepen the learning of quantum mechanics by their previous knowledge and experience gained from the course of engineering mechanics. The present book is the outgrowth of the lecture notes of this course. The success of the NCKU teaching program shows that complex mechanics shortens the gap between quantum mechanics and classical mechanics and makes the teaching and learning of quantum mechanics much easier.

Organization

All the materials developed in this book are based on the unique assumption that the real world is a place we take measurements of a physical motion, while the complex space is a place the physical motion actually takes place. Unfortunately, this assumption is not widely accepted in the contemporary physics and few references can be referred to during the development of complex mechanics. El Naschie's *E*-infinity theory is one of the very few publications that hold the same view as complex mechanics. Because of its independence from the mainstream physics, the book is written and organized to be self-contained and self-consistent. The book is constituted by five parts: (a) the basic assumptions and definitions of complex motions, (2) the fundamental principles derived from the assumptions and definitions, (3) the predictions and verifications of various quantum phenomena by the derived principles, (4) 3D complex motion in hydrogen atom, and (5) complex motion in 4D spacetime. The topics covered by the five parts and their interrelations are summarized in Fig. C.

(1) **Define physical motions in complex space**: Chapter 1

Chapter 1 surveys the methods and concepts needed in the book to describe physical motions in complex space, and introduces the necessary remedies for the conventional complex variable theory to cope with complex variables with internal dynamics.

(2) **Develop methods of analysis and mathematical tools**: Chapter 2 ~ Chapter 5

- Internal Dynamics of quantum states: In complex mechanics, physical quantities are complex variables with memory, i.e., they have internal dynamics, which can be used to trace their past motions and predict their future motions. Chapter 2 shows that internal dynamics is governed by deterministic Hamilton equations of motion defined in the complex space so that all the analytical methods developed in classic mechanics can be applied to investigate the internal dynamics of a quantum state.
- Complex representation of quantum operators: Chapter 3 points out that every quantum operator $\hat{\mathrm{A}}$ has a complex representation $\mathrm{A}(\mathbf{q},\mathbf{p})$ in complex mechanics. Knowing the expression for $\mathrm{A}(\mathbf{q},\mathbf{p})$ with $\mathbf{q}$ and $\mathbf{p}$ satisfying Hamilton equations of motion allows us to deduce $\hat{\mathrm{A}}$ readily. Through the equivalence between $\hat{\mathrm{A}}$ and $\mathrm{A}(\mathbf{q},\mathbf{p})$, the role of $\hat{\mathrm{A}}$ in quantum mechanics now can be replaced by the role of $\mathrm{A}(\mathbf{q},\mathbf{p})$ in complex mechanics.
- Quantization and time-averaged mean value: According to the time history of a physical quantity $f(x(t), p(t))$ solved from its internal dynamics, the time-averaged mean value $\langle f(x,p)\rangle_{x(t)}$ will be computed and compared to the probability-based mean value in Chapter 4. It is found that quantization of $f(x,p)$ is just the phenomenon that its time-averaged value $\langle f(x,p)\rangle_{x(t)}$ is independent of the trajectory $x(t)$ along which the time average is taken. Both local and global trajectory independences are discussed in Chapter 4. The former leads to the quantization of $\langle f(x,p)\rangle_{x(t)}$ in a given state ψ and the latter leads to the conservation of $\langle f(x,p)\rangle_{x(t)}$ in ψ.

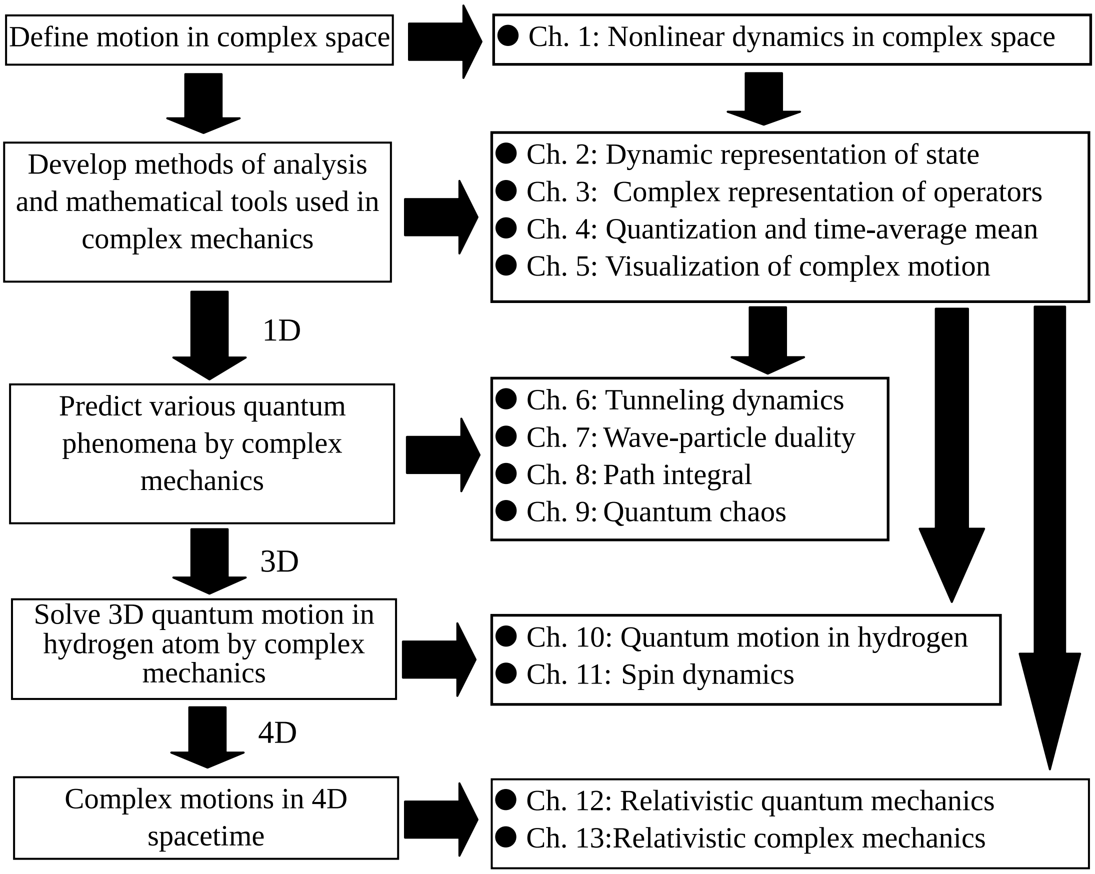

Fig. C. The topics covered by the book and their interrelations.

● Visualization of complex motions: Complex motion can be visualized by analogy with potential flows. Chapter 5 aims to reveal a novel analogy between probability flows and potential flows on the complex plane. For a given complex-valued wavefunction $\Psi(z,t)$, $z = x + iy \in \mathbb{C}$, we first define a complex potential function $\Omega(z,t) = \hbar/(im)\ln\Psi(z,t) = \phi(x,y,t) + i\psi(x,y,t)$ with $x, y \in \mathbb{R}$, and then prove that the streamline lines $\psi(x,y,t) = c_\psi$ and the potential lines $\phi(x,y,y) = c_\phi$ in the potential flow defined by Ω are equivalent to the constant-probability lines $|\Psi| = c_1$ and the constant-phase lines $\angle\Psi = c_2$ in the probability flow defined by Ψ. The discovered analogy is very useful in visualizing the unobservable probability flow on the complex $x + iy$ plane by analogy with the 2D potential flow on the real $x - y$ plane, which otherwise can be visualized by using dye streaks in a fluid laboratory.

(3) **Predict various quantum phenomena by complex mechanics**: Chapter 6 ~ Chapter 9

● Tunneling dynamics: Using the dynamic representation of a quantum state derived in Chapter 2, Chapter 6 models tunneling dynamics exactly by quantum Hamilton equations without any approximation. The advantage of solving tunneling problems by Hamilton mechanics is twofold. It makes the tunneling time as simple as the usual time without the necessity of defining any time operator and secondly, it provides the tunneling trajectory in an unambiguous way such that trajectories in classical regions and non-classical regions can be connected smoothly.

- Wave-particle duality: Wave motion associated with a material particle is produced by projecting its complex motion into real space. The aim of Chapter 7 is to verify this new interpretation of matter wave. The equations of motion for a free particle is solved therein to reveal how the interaction between real and imaginary motions can produce the particle's wave motion observed in real space.
- Feynman' path integral: Chapter 8 explains Feynman's path integral in terms of the multiple paths derived in Chapter 7. It is revealed that under the framework of complex mechanics path integral trajectories can be parameterized continuously in terms of a free parameter so that an infinite dimensional path integral can be transformed into a one-dimensional normal integral over this free parameter.
- Quantum chaos: Chapter 9 shows that the phenomena of quantum chaos is another kind of projection effects from the complex space to the real space. A new chaotic behavior, called strong chaos, is introduced in Chapter 9. Unlike classical chaos caused by the divergence of two trajectories emerging from two nearby initial positions, strong chaos is unique to quantum systems and is caused by the multi-path effect so that infinitely many trajectories may emerge spontaneously from the same initial position and diverge as time evolves.

(4) **Solve 3D quantum motion in hydrogen atom by complex mechanics**: Chapter 10, 11

- Complex motion in hydrogen atom: Chapter 10 studies electronic quantum dynamics in hydrogen atom by complex mechanics. We will see there that the quantizations of total energy and angular momentum are a natural consequence of the electron's complex motion. Meanwhile, the shell structure observed in the hydrogen atom is shown to be a manifestation of the quantum-potential structure, from which the quantum forces acting upon the electron can be uniquely determined, the stability of atomic configuration can be justified, and the electron's continuous transition from the quantum world to the classical world can be monitored as the quantum number $n \to \infty$.
- Complex representation of spin dynamics: The spin dynamics inherent in the Schrödinger equation has long been overlooked since the inception of quantum mechanics. Chapter 11 aims to report the discovery that the hydrogen ground-state wavefunctions solved from the Schrödinger equation without any correction from Pauli or Dirac theory clearly demonstrates the existence of an angular momentum $\pm\hbar/2$ when the orbital angular momentum L^2 is zero. The well-known spinless mode in the usual interpretation of the Schrödinger equation is only one of the three spin modes contained in the hydrogen ground states, while the remaining two modes inherent in the Schrödinger equation, namely, the spin-down mode with angular momentum $-\hbar/2$ and the spin-up mode with angular momentum $\hbar/2$, have never been reported before in the literature.

(5) **Extend complex motions to 4D spacetime**: Chapter 12, 13

- Relativistic quantum mechanics: Chapter 12 points out that extending special relativity to the complex spacetime automatically leads to the relativistic quantum mechanics. The complex spacetime is a bridge connecting the causality in special relativity to the non-locality in quantum mechanics. In the presence of quantum interactions, the correct mass-energy relation is found to be $E = mc^2\sqrt{1-2Q/(m_0c^2)}$. When the quantum potential Q is zero, it reduces to the famous result $E = mc^2$ in special relativity.
- Relativistic complex mechanics: In the final chapter, we will learn how to describe particle's complex motion in general curvilinear coordinates, subjected to electromagnetic field and

gravitational field. This generalization is helpful for us to extend complex mechanics to relativistic domain, wherein the space-time is curved and described by four-dimensional curvilinear coordinates. At the end of this chapter, we will see how classical mechanics, quantum mechanics and relativistic mechanics are unified under the framework of relativistic complex mechanics.

Fig. D. A Chinese couplet contrasts the yin-yang duality with the wave-particle duality via the connection of complex mechanics. Tai Chi considers our universe as an entanglement of Yin and Yang. Yin denoted by the black is the imaginary part of the universe and Yang denoted by the white is the real part of the universe. Motivated by Tai Chi, complex mechanics assumes that all physical quantities are complex variables having real parts as well as imaginary parts.

Philosophical Motivation from Tai Chi

The assumptions made in complex mechanics are based on the philosophy of Tai Chi, which advocates that the Nature (the *Tao*) contains two parts: *yang* is the observable (real) part and *yin* is the unobservable (imaginary) part. In the symbol of Tai Chi illustrated in Fig. D, yang is denoted by white and yin denoted by black so that within the black, there is always some white, and within the white, there is always some black. The figure of Tai Chi symbolizes the totality of the combined real and imaginary worlds and the entanglement between them. According to the philosophy of Tai Chi, the Nature is a complex-valued world, while what we sense and measure are only the real part of the world, which constitutes the physical world we experience in daily lives.

Complex mechanics is a scientific realization of Yin-Yang philosophy via the language of complex variables, providing a bridge between the Yin-Yang duality in Tai Chi and the wave-particle duality in quantum mechanics. The recent progress in complex mechanics has strengthened the fact that although the imaginary world cannot be directly sensed or measured in the real world, its influences on the Nature can be definitely detected via the measurement of its interaction with the real world. As will be demonstrated throughout this book, the coupling connection between the real and imaginary worlds gives rise to the various quantum phenomena that we have observed in the real world. The Chinese couplet shown in Fig. D highlights the role of the complex mechanics as a bridge between the Yin-Yang duality in Tai Chi and the wave-particle duality in quantum mechanics.

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Tainan, spring 2010 Ciann-Dong Yang