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Science is the moving boundary of innovation, and Nature is of nonlinear science.

D. D. Ganji, Seyed H. Hashemi Kachapi

Analysis of Nonlinear Equations in Fluids

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Editors

Ji-Huan He
Donghua University , Shanghai , China
Email: nlsaeditor@yahoo.cn

Sheng Zhang
Department of Mathematics
Bohai University
Jinzhou 121000, China
zhshaeng@yahoo.com.cn

Engui Fan
School of Mathematics, Fudan University,
Shanghai 204433,
P.R. China
Tel: 86-21-55665015
Fax: 86-21-65646073
E-mail: faneg@fudan.edu.cn

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Analysis of Nonlinear Equations in Fluids

Seyed H. Hashemi Kachapi
D. D. Ganji

Department of Mechanical Engineering
Babol Noshirvani University of Technology, P.O. Box 484, Babol, Iran
Email: sha.hashemi.kachapi@gmail.com, ddg_davood@yahoo.com

Abstract

Engineering is basically an application of mathematics and applied sciences to the solution of real world problems. In the majority of real-life and applied phenomena in engineering sciences and also other applied sciences in fluids field, solving to applied problems events are inevitable. In order to develop engineering sciences and applied sciences, it is necessary to carefully study analytical and numerical methods for solving of all available problems in case of linear and nonlinear equations. It is of great importance to study nonlinearity; because almost all applied processes act nonlinearly, and on the other hand, nonlinear analysis of complex systems is one of the most important and complicated tasks, especially in engineering and applied sciences problems.

The most important and fundamental step to analyze an engineering problem is to determine the equations governing the motion and dynamics of the problem unless investigating the problems is impossible. Since the equations governing the motion of the body or system determine the nature of its analysis, obtaining these the equations is of great importance. The equations governing the motion lead to the formation of ordinary or partially differential equations and different types of linear and especially nonlinear equations in general. Therefore in this book first some fundamental ways of obtaining the governing equations are introduced along with applied examples and in the following methods solve them are explained. © 2011 Asian Academic Publisher Limited. All rights reserved.

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Preface

Engineering is basically an application of mathematics and applied sciences to the solution of real world problems. In the majority of real-life and applied phenomena in engineering sciences and also other applied sciences in fluids field, solving to applied problems events are inevitable. In order to develop engineering sciences and applied sciences, it is necessary to carefully study analytical and numerical methods for solving of all available problems in case of linear and nonlinear equations. It is of great importance to study nonlinearity; because almost all applied processes act nonlinearly, and on the other hand, nonlinear analysis of complex systems is one of the most important and complicated tasks, especially in engineering and applied sciences problems.

None of the books in this area have completely studied and analyzed all applied processes in both linear and especially nonlinear forms, so that the user can solve the problems without the need of studying too many different references. Thereby in this book, by the use of the latest analytic, numeric laboratorial methods and using more than 300 references like books, papers and the researches done by the authors and by considering almost all possible processes and situation, new theories has been proposed to encounter applied problems in engineering and applied sciences. In this way, the user (bachelor's, master's and PhD students, university teachers and even in research centers in different fields of mechanical, civil, aerospace, electrical, chemical, applied mathematics, physics, and etc.) can encounter such systems in confidently. In the different chapters of the book, not only are the linear and nonlinear problems broadly discussed, but also applied examples are practically solved by the proposed methodology.

The users of this collection can achieve very strong capabilities in the area, especially in linear nonlinear phenomena, such as:

- A complete understanding of the formulation of motion equation in different systems using most general methods.
- A complete understanding of the fundamentals in analytical methods in solving to applied problems, especially nonlinearly.

A complete study of mathematical problems, analytic and numeric methods (e.g. Perturbation methods, Homotopy perturbation method, Iteration perturbation method and its Modified,

Variational methods, Energy methods, Phase plan, Limit cycle, Homotopy Analysis Method, Adomian's Decomposition Method, Differential Transformation Method and its Modified, Coupled Method of Homotopy Perturbation Method and Variational Method, Exp –Function Method, F-Expansion Method, Sine-Cosine Function Method, Tanh-Coth Method, (G'/G) -Expansion Method, and etc).

- Complete familiarity with specialized processes and applications in different areas of the field, studying them, elimination of complexities and controlling them, and also applying them in real-life engineering cases.
- Complete analysis of important engineering systems (e.g. NDOF systems, disc, multi-body phenomena, wave, heat and mass transfer, combustion, thermal, fluid flow, and etc).
- A complete analysis of important equation in the field and their generalization in real-life applications with practical examples (viscoelastic, quantum mechanics, fraction order, porous media equation, cubic nonlinearity, coupled system, reaction-diffusion equation, boundary value problems, convective-radiative heat transfers, Reynolds number, Blasius problem, wave equation and etc).
- The ability to encounter, model and interpret an engineering process or system, and to solve the related complexities.

Notation and Units

Both the SI and the US/English system of units have been used throughout the book.

Audience

This book is a comprehensive and complete text on analytical methods in applied problems. It is self-contained and the subject matter is presented in an organized and systematic manner. This book is quite appropriate for several groups of people including:

- Senior undergraduate and graduate students taking the course analytical methods.
- The book can be adapted for a short professional course on the subject matter.
- Design and research engineers will be able to draw upon the book in selecting and developing mathematical models for analytical and design purposes in applied conditions.
- Practicing engineers and managers who want to learn about the basic principles and concepts involved in solving of problems using analytical methods such fluid flow, systems analysis and how it can be applied at their own work place concerns.
- Generally, the user are bachelor's, master's and PhD students, university teachers and even in research centers in different fields of mechanical, civil, aerospace engineering, etc.

Because the book is aimed at a wider audience, the level of mathematics is kept intentionally low. All the principles presented in the book are illustrated by numerous worked examples. The book draws a balance between theory and practice.

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I would appreciate being informed of errors, or receiving other comments about the book. Please write to the Authors' Babol Noshirvani University of Technology address or send e-mail to:

sha.hashemi.kachapi@gmail.com

ddg_davood@yahoo.com

We sincerely hope that the final outcome of this book helps the students, researcher and other user in developing an appreciation for the topic of analytical and numerical methods.

Seyed H. Hashemi Kachapi

D. D. Ganji