

# **Progress in Nonlinear Science**



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Science is the moving boundary of innovation, and Nature is of nonlinear science.



D. D. Ganji, Seyed H. Hashemi Kachapi

Analytical and Numerical Methods in Engineering  
and Applied Sciences

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## **Analytical and Numerical Methods in Engineering and applied Sciences**

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### **Abstract**

Engineering is basically an application of mathematics and applied sciences to the solution of real world problems. In the majority of real-life and applied phenomena in engineering sciences and also other applied sciences, solving to applied problems events are inevitable. In order to develop engineering sciences and applied sciences, it is necessary to carefully study analytical and numerical methods for solving of all available problems in case of linear and nonlinear equations. It is of great importance to study nonlinearity; because almost all applied processes act nonlinearly, and on the other hand, nonlinear analysis of complex systems is one of the most important and complicated tasks, especially in engineering and applied sciences problems.

The most important and fundamental step to analyze an engineering problem is to determine the equations governing the motion and dynamics of the problem unless investigating the problems is impossible. Since the equations governing the motion of the body or system determine the nature of its analysis, obtaining these the equations is of great importance. The equations governing the motion lead to the formation of ordinary or partially differential equations and different types of linear and nonlinear equations in general. Therefore in this book first some fundamental methods of obtaining the governing equations are introduced along with applied examples and in the following methods solve them are explained. © 2011 Asian Academic Publisher Limited. All rights reserved.

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# Preface

Engineering is basically an application of mathematics and applied sciences to the solution of real world problems. In the majority of real-life and applied phenomena in engineering sciences and also other applied sciences, solving to applied problems events are inevitable. In order to develop engineering sciences and applied sciences, it is necessary to carefully study analytical and numerical methods for solving of all available problems in case of linear and nonlinear equations. It is of great importance to study nonlinearity; because almost all applied processes act nonlinearly, and on the other hand, nonlinear analysis of complex systems is one of the most important and complicated tasks, especially in engineering and applied sciences problems.

None of the books in this area have completely studied and analyzed all applied processes in both linear and nonlinear forms, so that the user can solve the problems without the need of studying too many different references. Thereby in this book, by the use of the latest analytic, numeric laboratorial methods and using more than 300 references like books, papers and the researches done by the author and by considering almost all possible processes and situation, new theories has been proposed to encounter applied problems in engineering and applied sciences. In this way, the user (bachelor's, master's and PhD students, university teachers and even in research centers in different fields of mechanical, civil, electrical, chemical, applied mathematics, physics, and etc.) can encounter such systems in confidently. In the different chapters of the book, not only are the linear and nonlinear problems broadly discussed, but also applied examples are practically solved by the proposed methodology.

Abundant number of examples and homework problems is provided.

The users of this collection can achieve very strong capabilities in the area, especially in linear nonlinear phenomena, such as:

- A complete understanding of the fundamentals in analytical and numerical methods in solving to applied problems, especially nonlinearly.
- A complete study of mathematical problems, analytic and numeric methods (e.g. Periodic solutions, Perturbation methods, Homotopy perturbation method, Iteration perturbation method, Variational methods, Energy methods, Principle of Virtual Work, D'Alembert's Principle, Energy Method, Newton and Lagrange methods, Stability, Phase plan, Limit cycle, Modulation Equations, Dunkerley's method, Rayleigh's Method, Rayleigh-Ritz Method, Holzer Method, Jacobi Diagonalization Method, Cholesky Decomposition Method,

Iteration Methods, Matrix Iteration Method, Inverse Matrix Method, Simultaneous Iteration Method, Subspace Iteration Method, Finite Difference Method, Central Difference Method, Runge - Kutta Method, Houbolt Method, Wilson Theta Method, Newmark Beta Method, Park Stiffly Stable Method, Fourier Series, Laplace Transforms and etc)

- Complete familiarity with specialized processes and applications in different areas of the field, studying them, elimination of complexities and controlling them, and also applying them in real-life engineering cases.
- Complete analysis of important engineering systems (e.g. Gear Box, NDOF systems, Robot, Disc, Normal Modes, Multi-body phenomena, Shaft, Slider, etc).
- A complete analysis of important equation in the field and their generalization in real-life applications with practical examples (Duffing oscillation, Van der Pol oscillation, Mathieu oscillation, Hamiltonian).
- The ability to encounter, model and interpret an engineering process or system, and to solve the related complexities.

## **Notation and Units**

Both the SI and the US/English system of units have been used throughout the book.

## **Audience**

This book is a comprehensive and complete text on analytical and numerical methods in applied problems. It is self-contained and the subject matter is presented in an organized and systematic manner. This book is quite appropriate for several groups of people including:

- Senior undergraduate and graduate students taking the course analytical and numerical methods.
- The book can be adapted for a short professional course on the subject matter.
- Design and research engineers will be able to draw upon the book in selecting and developing mathematical models for analytical and design purposes in applied conditions.
- Practicing engineers and managers who want to learn about the basic principles and concepts involved in solving of problems using analytical and numerical methods such dynamic, vibration, oscillation systems analysis and how it can be applied at their own work place concerns.
- Generally, the user are bachelor's, master's and PhD students, university teachers and even in research centers in different fields of mechanical, civil, aerospace engineering, etc.

Because the book is aimed at a wider audience, the level of mathematics is kept intentionally low. All the principles presented in the book are illustrated by numerous worked examples. The book draws a balance between theory and practice.

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