Nonlinear Science Series

Xiao-Jun Yang

Local Fractional Functional Analysis & Its Applications

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非线性科学系列

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Nonlinear Science Series focuses on recent advances of fundamental theories and principles, analytical and numerical methods in nonlinear science with engineering applications.

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This Book is dedicated to my parents for supporting me in all my endeavors.

Xiao-Jun Yang

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Preface

This book briefly introduces local fractional functional analysis and its applications to fractal integral transforms and fractal wavelets. It is a good tool for us to do fractal mathematics and engineering with the operators of local fractional derivatives and local fractional integrals. This book attempts to apply knowledge on this topic in the field of fractal mathematics science and fractal engineering and gives a new and mathematically rigorous account of the integral transforms containing Yang-Fourier transforms, Yang-Laplace transforms, local fractional short-time continuous transforms and local fractional wavelets transforms. This book is divided into a number of parts.

Chapter 1 provides basic theory of real line number on fractional sets and proposes the local fractional continuity of non-differentiable functions. The real and complex spaces on fractional sets, the complex conjugation, absolute value, and polar form of a fractional complex numbers are proposed and geometric representation of real line number on fractional sets is discussed. Furthermore, generalized Lebesgue measure, point sets, intervals, countability, neighborhood, limit point, limit of a fractional sequence, bounds, generalized Hausdorff measure and functions on fractional sets are derived. Finally, the theory of local fractional continuity of non-differential functions is discussed and some basic functions on fractional sets are discussed.

In chapter 2 the fundamentals of the local fractional calculus of real variables are outlined. The chapter begins with the definition of local fractional derivatives and the elementary theory of local fractional derivatives of non-differential functions. Existence of the local fractional integrals is proved and the basic properties and theorems of the local fractional integrals are discussed. To study local fractional differential equations, the concept of the local fractional indefinite integrals is derived. To investigate the total local fractional differentials, local fractional partial derivative, local fractional derivative of high order, and local fractional Jacobian determinant are introduced. Finally, Yang-Taylor series for two-variable functions is obtained.

Chapter 3 introduces the fundamentals of the local fractional calculus of complex variables. With the chapter starts by deriving limit and local fractional continuity of complex functions on fractional sets, the local fractional derivatives, local fractional Cauchy-Riemann equations and local fractional integrals of complex functions. The local fractional Cauchy integrals of complex functions and local fractional Taylor's series (also called Yang-Taylor series) and local fractional Laurent's series are also discussed. Lastly, the generalized residue theorems are mentioned.   A short outline of local fractional complex analysis is proposed in this chapter.

Chapter 4 derives generalized fractal spaces, such as generalized metric spaces, generalized normed linear spaces, generalized Banach spaces, generalized inner product spaces, and generalized Hilbert spaces. Based on above definitions, we present the completeness of generalized fractal spaces, and extend contracting mapping theorem and generalized contracting mapping theorem, existence and unique of local fractional differential equation, Banach algebra, Pythagorean theorem and the basic criterion for generalized Hilbert spaces to fractional spaces. We obtain the generalized Holder inequality, the generalized Cauchy-schwarz inequality and the generalized Minkowski inequality and some spaces are discussed.

Chapter 5 derives the local fractional series containing fractional trigonometric and Mattag-Leffter forms. Meanwhile, the properties and theorems of the local fractional series are discussed. Finally, a typical application of local fractional Fourier series to local fractional partial differential equation is discussed.

Chapter 6 introduces the Yang-Fourier transforms derive from local fractional series based on the local fractional calculus. Meanwhile, the properties and theorems for Yang-Fourier transforms are discussed. In addition, Heisenberg uncertainty principles in fractal spaces are investigated. Applications of the Yang-Fourier transforms to local fractional ordinary differential equations and local fractional ordinary differential systems are taken into account. Finally, generalized Yang-Fourier transforms are derived from local fractional calculus and special applications of Yang-Fourier transforms are discussed.

Chapter 7 introduces the Yang-Laplace transforms derived from Yang-Fourier transforms. We derive the properties and theorems for the Yang-Laplace transforms and take into account its applications to local fractional ordinary differential equations, local fractional ordinary differential systems and local fractional partial differential equations.

Chapter 8 studies the local fractional short time transforms, and properties and examples of possible applications.

Chapter 9 derives local fractional continuous wavelet transforms based on Yang-Fourier transforms and theorems for the local fractional continuous wavelet transforms.

This book keeps structure of the journal-like book *Local Fractional Integral Transforms*, and we add some recent results.

In this book the various transformations are derived step by step in great detail. We hope this book will be a useful tool for all those who use local fractional integral transforms and local fractional continuous wavelet transform in their work whether they are engineers, financial planners, mathematicians and scientists. The book can also provide a first course on an introduction to local fractional functional analysis and its applications to fractal integral transforms and fractal wavelets. Several mistakes and misprints were pointed out to us by a number of people and had been corrected. We want to thank those people for their helpful comments. We welcome comments from our readers.

Contents

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Local fractional functional analysis is a totally new area of mathematics, and a totally new mathematical world view as well. In this book, a new approach to functional analysis on fractal spaces, which can be used to interpret fractal mathematics and fractal engineering, is presented. From Cantor sets to fractional sets, real line number and the spaces of local fractional functions are derived. Local fractional calculus of real and complex variables is systematically elucidated. Some generalized spaces, such as generalized metric spaces, generalized normed linear spaces, generalized Banach's spaces, generalized inner product spaces and generalized Hilbert spaces, are introduced. Elemental introduction to the Yang-Fourier transform, the Yang-Laplace transform, the local fractional short time transform and the local fractional continuous wavelet transform is presented based on the generalized fractal spaces.

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